1 UNITS AND MEASUREMENTS

1. PHYSICAL QUANTITIES

	♦	PHYSICAL QUANTITY UNITS
	♦	FUNDAMENTAL AND DERIVED UNITS

PHYSICAL QUANTITY

A quantity which can be measured and by which various physical happenings can be explained and expressed in form of laws is called a physical quantity. For example, length, mass, time, force etc.

On the other hand, various happenings in life e.g., happiness, sorrow etc. are not physical quantities because these cannot be measured.

Measurement is necessary to determine magnitude of a physical quantity, to compare two similar physical quantities and to prove physical laws or equations.

A physical quantity is represented completely by its magnitude and unit. For example, 10 metre means a length which is ten times the unit of length 1 kg. Here 10 represents the numerical value of the given quantity and metre represents the unit of quantity under consideration. Thus, in expressing a physical quantity we choose a unit and then find that how many times that unit is contained in the given physical quantity, i.e.

Physical quantity (Q) = Magnitude × Unit = n × u

Where, n represents the numerical value and u represents the unit. Thus, while expressing definite amount of physical quantity, it is clear that as the unit(u) changes, the magnitude(n) will also change but product 'nu' will remain same.

i.e., nu = constant, or $n_1u_1 = n_2u_2 = \text{constant}$; $\therefore n \propto \frac{1}{u}$

i.e., magnitude of a physical quantity and units are inversely proportional to each other. Larger the unit, smaller will be the magnitude.

TYPES OF PHYSICAL QUANTITY

(1) **Ratio (numerical value only):** When a physical quantity is a ratio of two similar quantities, it has no unit.

Ex: Relative density = Density of object/Density of water at 4ºC Refractive index = Velocity of light in air/Velocity of light in medium

Strain = Change in dimension/Original dimension

Note: Angle is exceptional physical quantity, which though is a ratio of two similar physical quantities (angle = arc / radius) but still requires a unit (degrees or radians) to specify it along with its numerical value.

(2) **Scalar (Magnitude only):** These quantities do not have any direction e.g. Length, time, work, energy etc.

Magnitude of a physical quantity can be negative. In that case negative sign indicates that the numerical value of the quantity under consideration is negative. It does not specify the direction.

Scalar quantities can be added or subtracted with the help of following ordinary laws of addition or subtraction.

(3) **Vector (magnitude and direction):** Ex: displacement, velocity, acceleration, force etc.

Vector physical quantities can be added or subtracted according to vector laws of addition. These laws are different from laws of ordinary addition.

Note: There are certain physical quantities which behave neither as scalar nor as vector. For example, moment of inertia is not a vector as by changing the sense of rotation its value is not changed. It is also not a scalar as it has different values in different directions (i.e. about different axes). Such physical quantities are called Tensors.

FUNDAMENTAL AND DERIVED QUANTITIES

(1) **Fundamental quantities:** Out of large number of physical quantities which exist in nature, there are only few quantities which are independent of all other quantities and do not require the help of any other physical quantity for their definition, therefore these are called absolute quantities. These quantities are also called fundamental or base quantities, as all other quantities are based upon and can be expressed in terms of these quantities.

(2) **Derived quantities:** All other physical quantities can be derived by suitable multiplication or division of different powers of fundamental quantities. These are therefore called derived quantities.

If length is defined as a fundamental quantity, then area and volume are derived from length and are expressed in term of length with power 2 and 3 over the term of length.

Note: In mechanics Length, Mass and time are arbitrarily chosen as fundamental quantities. However, this set of fundamental quantities is not a unique choice. In fact any three quantities in mechanics can be termed as fundamental as all other quantities in mechanics can be expressed in terms of these. e.g. if speed and time are taken as fundamental quantities, length will become a derived quantity because then length will be expressed as Speed ×Time. and if force and acceleration are taken as fundamental quantities, then mass will be defined as Force / acceleration and will be termed as a derived quantity.

FUNDAMENTAL AND DERIVED UNITS

Normally each physical quantity requires a unit or standard for its specification so it appears that there must be as many units as there are physical quantities. However, it is not so. It has been found that if in mechanics we choose arbitrarily units of any three physical quantities we can express the units of all other physical quantities in mechanics in terms of these. Arbitrarily the physical quantities mass, length and time are chosen for this purpose. So, any unit of mass, length and time in mechanics is called a **fundamental, absolute or base unit.** Other units which can be expressed in terms of fundamental units, are called derived units. For example, light year or km is a fundamental unit as it is a unit of length while s^{-1}, m^2 or kg/m are derived units as these are derived from units of time, mass and length respectively.

System of units: A complete set of units, both fundamental and derived for all kinds of physical quantities is called system of units. The common systems are given below

(1) **CGS system:** The system is also called Gaussian system of units. In it length, mass and time have been chosen as the fundamental quantities and corresponding fundamental units are centimetre (cm), gram (g) and second (s) respectively.

(2) **MKS system:** The system is also called Giorgi system. In this system also length, mass and time have been taken as fundamental quantities, and the corresponding fundamental units are metre, kilogram and second.

(3) **FPS system:** In this system foot, pound and second are used respectively for measurements of length, mass and time. In this system force is a derived quantity with unit poundal.

(4) **SI system:** It is known as international system of units and is infact extended system of units applied to whole physics. There are seven fundamental quantities in this system. These quantities and their units are given in the following table.

Quantity	Name of Unit	Symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric Current	ampere	A
Temperature	Kelvin	K
Amount of Substance	mole	mol
Luminous Intensity	candela	cd

Besides the above seven fundamental units two supplementary units are also defined - Radian (rad) for plane angle and Steradian (sr) for solid angle.

Note: Apart from fundamental and derived units we also use very frequently practical units. These may be fundamental or derived units.

Ex: light year is a practical unit (fundamental) of distance while horsepower is a practical unit (derived) of power.

Practical units may or may not belong to a system but can be expressed in any system of units.

Ex: 1 mile = 1.6 km = 1.6×10^3 m.

MEASUREMENT OF LENGTH

The length of an object can be measured by using different units. Some particle units of length are angstrom $(A^o) = 10^{-10} m = 10^{-8} cm$

Nanometer $(nm) = 10^{-9} m = 10 A^0$

Fermi $= 10^{-15} m$

Micron $= 10^{-6} m$

X-ray unit $= 10^{-13} m$

1 A.U. = distance between sun & earth

$= 1.496 \times 10^{11}$ m

One light year is the distance travelled by light in one year in vacuum. This unit is used in astronomy.

Light year $= 9.46 \times 10^{15} m$

parsec = 3.26; light years $= 30.84 \times 10^{15} m$

Bohr radius $= 0.5 \times 10^{-10} m$

Mile=1.6 km

Measurement of mass:

The mass of an object can be measured by using different units. Some practical units of mass are

Quintal = 100 kg

Metric ton = 1000 kg

Atomic mass unit (a.m.u) $= 1.67 \times 10^{-27} kg$

Measurement of time:

One day = 86400 second

Shake $= 10^{-8}$ second

Abbreviations for multiples and sub multiples:

MACRO Prefixes

Multiplier	Symbol	Prefix
10^1	da	Deca
10^2	h	Hecto
10^3	k	Kilo
10^6	M	Mega
10^9	G	Giga
10^{12}	T	Tera
10^{15}	P	Peta
10^{18}	E	Exa
10^{21}	Z	Zetta
10^{24}	Y	Yotta

MICRO Prefixes

Multiplier	Symbol	Prefix
10^{-1}	d	deci
10^{-2}	c	centi
10^{-3}	m	milli
10^{-6}	μ	micro
10^{-9}	n	nano
10^{-12}	p	pico
10^{-15}	f	femto
10^{-18}	a	atto
10^{-21}	z	zepto
10^{-24}	y	yocto

Some important conversions:

$1\text{kmph}=\frac{5}{18}\text{ms}^{-1}$

1 newton= 10^5 dyne

1 joule=10^7 erg

1 calorie=4.18 J

$1\text{gcm}^{-3}=1000\text{kgm}^{-3}$

1 litre=$1000\text{cm}^3=10^{-3}\text{m}^3$

$1\text{KWH}=36\times10^5\text{J}$

1 HP=746 W

1 degree=0.017 rad

1kgwt= 9.8 N

1 telsa= 10^4 gauss

1 weber=10^8 Maxwell

Some physical constants and their values:

1 amu $=1.67\times10^{-27}$kg=931.5MeV

1 atm pressure = pressure exerted by 76cm of Hg column$=1.013\times10^5$Pa

Permittivity of free space=8.854×10^{-12}Fm^{-1} or C^2/Nm^2

Permeability of free space $(\mu_0)=4\pi\times10^{-7}Hm^{-1}$

Joule's constant (J)=$4.186Jcal^{-1}$

Planck's constant(h)= 6.62×10^{-34}Js

Universal gas constant(R)=$8.314Jmol^{-1}K^{-1}$

$=1.98cal\,mol^{-1}K^{-1}$

Wien's constant(b)=2.93×10^{-3}metre kelvin

1. PHYSICAL QUANTITIES

WORK SHEET LEVEL–I MAINS CORNER

SINGLE CORRECT ANSWER TYPE QUESTIONS

PHYSICAL QUANTITY UNITS

1. Which of the following systems of units is not based on units of mass, length and time alone?
   1) SI 2) MKS 3) CGS 4) FPS
2. The magnitude of any physical quantity
   1) Depends on the method of measurement
   2) Does not depend on the method of measurement
   3)Is more in SI system than in CGS system
   4) Directly proportional to the fundamental units of mass, length and time
3. Newton–second is the unit of
   1)Velocity 2) Angular momentum
   3)Momentum 4) Energy
4. Which of the following is not represented in correct unit
   1) $\frac{\text{Stress}}{\text{Strain}} = \text{N/m}^2$ 2) Surface tension =N/m
   3)Energy $=$ kg-m/sec 4) Pressure $= \text{N/m}^2$
5. Joule-second is the unit of
   1) Work 2) Momentum
   3) Pressure 4) Angular momentum
6. Unit of power is
   1) Kilowatt 2) Kilowatt-hour 3) Dyne 4) Joule
7. A suitable unit for gravitational constant is
   1) kg-m sec^{-1} 2) $\text{N m}^{-1}\text{sec}$ 3) $\text{N m}^2\text{kg}^{-2}$ 4) kg m sec^{-1}
8. Which of the following is a derived unit?
   1) Unit of mass 2) Unit of length
   3) Unit of time 4) Unit of volume

LEVEL–II

PHYSICAL QUANTITY UNITS

9. One nanometre is equal to
   1)10^9 mm 2)10^{-6} cm 3)10^{-7} cm 4)10^{-9} cm
10. A micron is related to centimetre as
    1)1micron $= 10^{-8}$ cm 2)1micron $= 10^{-6}$ cm
    3)1micron $= 10^{-5}$ cm 4)1micron $= 10^{-4}$ cm

11. In $S = a + bt + ct^2$. S is measured in metre and t in seconds. The unit of 'c' is
1) None 2) m 3) ms^{-1} 4) ms^{-2}

12. Density of wood is $0.5 gm / cc$ in the CGS system of units. Then $\frac{1}{100^{th}}$ of the corresponding value in MKS units is
1) 500 2) 5 3) 0.5 4) 5000

13. One million electron volt $(1MeV)$ is equal to
1) $10^5 eV$ 2) $10^6 eV$ 3) $10^4 eV$ 4) $10^7 eV$

14. The unit of potential energy is
1) $g(cm/s^2)$ 2) $g(cm/s)^2$ 3) $g(cm^2/s)$ 4) $g(cm/s)$

15. newton/metre^2 is the unit of
1) Energy 2) Momentum 3) Force 4) Pressure

LEVEL-III ADVANCED CORNER

SINGLE CORRECT ANSWER TYPE QUESTIONS

16. The velocity of a particle depends upon as $v = a + bt + ct^2$; if the velocity is in m / sec, the unit of a will be
1) m/s 2) m/s^2 3) m^2/s 4) m/s^3

17. Parsec is a unit of
1) Distance 2) Velocity 3) Time 4) Angle

18. If u_1 and u_2 are the units selected in two systems of measurement and n_1 and n_2 their numerical values, then
1) $n_1u_1 = n_2u_2$ 2) $n_1u_1 + n_2u_2 = 0$
3) $n_1n_2 = u_1u_2$ 4) $(n_1 + u_1) = (n_2 + u_2)$

19. $1 kWh =$
1) $1000 W$ 2) $36 \times 10^5 J$ 3) $1000 J$ 4) $3600 J$

20. The value of Planck's constant is
1) 6.63×10^{-34} J-s 2) 6.63×10^{34} J/s
3) 6.63×10^{-34} kg-m^2 4) 6.63×10^{34} kg/s

21. A physical quantity is measured and its value is found to be nu where n= numerical value and u=unit. Then which of the following relations is true
1) $n \propto u^2$ 2) $n \propto u$ 3) $n \propto \sqrt{u}$ 4) $n \propto \frac{1}{u}$

22. Unit of impulse is
1) Newton 2) kg–m 3) kg–m/s 4) Joule

23. One astronomical unit is a distance equal to
1) 9.46×10^{15} m 2) 1.496×10^{11} m 3) 3×10^{8} m 4) 3.08×10^{16} m

LEVEL–IV

STATEMENT TYPE QUESTIONS

24. Statement I: Light year and year, both measure time.
Statement II: Light year is the distance which light covers in one year.
1) Both statements are true
2) Both statements are false
3) Statement I is true, statement II is false
4) Statement I is false, statement II is true

MULTI CORRECT ANSWER TYPE QUESTIONS

25. Which of the following are correct SI units?
1) Meter for length
2) Kilogram for mass
3) Kelvin for temperature
4) Minute for time

26. Which of the following are not a unit of time?
1) Parsec 2) Light year 3) Micron 4) Second

LEVEL–V

COMPREHENSION TYPE QUESTIONS

PASSAGE:

Derived units are the units of derived physical quantities which are expressed in terms of fundamental units

27. Kilowatt hour is the unit of
1) Power 2) Energy 3) Pressure 4) Time

28. Pascal is the S.I unit of
1) Impulse 2) Coefficient of viscosity
3) Surface tension 4) Modulus of elasticity

29. $Kgms^{-1}$ is the unit of
1) Force 2) Linear momentum
3) angular momentum 4) Torque

MATRIX MATCH TYPE QUESTIONS

30.

Column-I	Column-II
a) Practical unit	p) radian
b) Fundamental unit	q) Light year
c) Derived unit	r) $kg\text{-}ms^{-1}$
d) Supplementary unit	s) Second

2. DIMENSIONS OF A PHYSICAL QUANTITY

	♦	DIMENSIONAL FORMULA
	♦	APPLICATIONS OF DIMENSIONAL ANALYSIS

DIMENSIONS OF A PHYSICAL QUANTITY

When a derived quantity is expressed in terms of fundamental quantities, it is written as a product of different powers of the fundamental quantities. The powers to which fundamental quantities must be raised in order to express the given physical quantity are called its dimensions.

To make it more clear, consider the physical quantity force

$$\text{Force} = \text{mass} \times \text{acceleration} = \frac{\text{mass} \times \text{velocity}}{\text{time}}$$

$$= \frac{\text{mass} \times \text{length/time}}{\text{time}} = \text{mass} \times \text{length} \times (\text{time})^{-2} \qquad \text{.... (i)}$$

Thus, the dimensions of force are 1 in mass, 1 in length and – 2 in time.

Here the physical quantity that is expressed in terms of the base quantities is enclosed in square brackets to indicate that the equation is among the dimensions and not among the magnitudes.

Thus equation (i) can be written as [force] = $[MLT^{-2}]$.

Such an expression for a physical quantity in terms of the fundamental quantities is called the dimensional equation. If we consider only the R.H.S. of the equation, the expression is termed as dimensional formula.

Thus, dimensional formula for force is, $[MLT^{-2}]$.

IMPORTANT DIMENSIONS OF COMPLETE PHYSICS

Mechanics

S. N.	Quantity	Relation with other quantities	Unit	Dimension
(1)	Velocity or speed (v)	$\frac{\text{displacement}}{\text{time}}$	m/s	$[M^0L^1T^{-1}]$
(2)	Acceleration (a)	$\frac{\Delta v}{\Delta t}$	m/s^2	$[M^0LT^{-2}]$
(3)	Momentum (P)	P=mv	kg-m/s	$[M^1L^1T^{-1}]$
(4)	Impulse (J)	J=F.Δt	Newton-second or kg-m/s	$[M^1L^1T^{-1}]$
(5)	Force (F)	F=ma	Newton	$[M^1L^1T^{-2}]$

S. N.	Quantity	Relation with other quantities	Unit	Dimension
(6)	Pressure (P)	$p=\frac{F}{A}$	Pascal N/m^2	$[M^1L^{-1}T^{-2}]$
(7)	Kinetic energy (E_K)	$K=\frac{1}{2}mv^2$	Joule	$[M^1L^2T^{-2}]$
(8)	Power (P)	$P=\frac{W}{t}$	Watt or J/s	$[M^1L^2T^{-3}]$
(9)	Density (d)	$d=\frac{M}{V}$	kg/m^3	$[M^1L^{-3}T^0]$
(10)	Angular displacement (θ)		Radian (rad.)	$[M^0L^0T^0]$
(11)	Angular velocity (ω)	$\omega=\frac{\Delta\theta}{\Delta t}$	Radian/second	$[M^0L^0T^{-1}]$
(12)	Angular acceleration (α)		Radian/second^2	$[M^0L^0T^{-2}]$
(13)	Moment of inertia (I)		kg-m^2	$[M^1L^2T^0]$
(14)	Torque (τ)	$\tau=F\times r$	Newton-meter	$[M^1L^2T^{-2}]$
(15)	Angular momentum (L)	$L=I\omega$	Joule-second	$[M^1L^2T^{-1}]$
(16)	Force constant or spring constant (k)		Newton/m	$[M^1L^0T^{-2}]$
(17)	Gravitational constant (G)	$G=\frac{Fr^2}{m_1m_2}$	N-m^2/kg^2	$[M^{-1}L^3T^{-2}]$
(18)	Intensity of gravitational field (E_g)	$\left[\frac{F}{M}\right]$	N/kg	$[M^0L^1T^{-2}]$
(19)	Gravitational potential (V_g)		Joule/kg	$[M^0L^2T^{-2}]$
(20)	Surface tension (T)	$S=\frac{F}{l}$	N/m or J/m^2	$[M^1L^0T^{-2}]$
(21)	Velocity gradient (V_g)	$\left[\frac{dV}{dx}\right]$	Second^{-1}	$[M^0L^0T^{-1}]$
(22)	Coefficient of viscosity (η)	$\eta=\frac{F}{6\pi rv}$	kg/m-s	$[M^1L^{-1}T^{-1}]$

S. N.	Quantity	Relation with other quantities	Unit	Dimension
(23)	Stress	$\sigma=\frac{F}{A}$	N/m^2	$[M^1L^{-1}T^{-2}]$
(24)	Strain	$\in=\frac{\Delta l}{l}$	No unit	$[M^0L^0T^0]$
(25)	Modulus of elasticity (E)	$Y=\frac{\sigma}{\in}=\frac{\text{stress}}{\text{strain}}$	N/m^2	$[M^1L^{-1}T^{-2}]$
(26)	Poisson Ratio (σ)	$\frac{\text{lateral strain}}{\text{longitudinal strain}}$	No unit	$[M^0L^0T^0]$
(27)	Time period (T)		Second	$[M^0L^0T^1]$
(28)	Frequency (n)	$\frac{1}{T}$	Hz	$[M^0L^0T^{-1}]$

Heat

S. N.	Quantity	Relation with other quantities	Unit	Dimension
(29)	Temperature (T)		Kelvin	$[M^0L^0T^0\theta^1]$
(30)	Heat (Q)		Joule	$[ML^2T^{-2}]$
(31)	Specific Heat (S)	$S=\frac{Q}{m\Delta\theta}$	Joule/kg-K	$[M^0L^2T^{-2}\theta^{-1}]$
(32)	Thermal capacity	$C=ms=\frac{Q}{\Delta\theta}$	Joule/K	$[M^1L^2T^{-2}\theta^{-1}]$
(33)	Latent heat (L)	$L=\frac{Q}{m}$	Joule/kg	$[M^0L^2T^{-2}]$
(34)	Gas constant (R)	$R=\frac{PV}{nT}$	Joule/mol-K	$[M^1L^2T^{-2}\theta^{-1}]$
(35)	Boltzmann constant (k)	$K=\frac{R}{N_A}$	Joule/K	$[M^1L^2T^{-2}\theta^{-1}]$
(36)	Coefficient of thermal conductivity (K)		Joule/m-s-K	$[M^1L^1T^{-3}\theta^{-1}]$
(37)	Stefan's constant (σ)	$\sigma=\frac{P}{AT^4}$	$Watt/m^2$-K^4	$[M^1L^0T^{-3}\theta^{-4}]$
(38)	Wien's constant (b)		Meter-K	$[M^0L^1T^o\theta^1]$

S. N.	Quantity	Relation with other quantities	Unit	Dimension
(39)	Planck's constant (h)	$h=\frac{E}{v}$	Joule-second	$[M^1L^2T^{-1}]$
(40)	Coefficient of Linear Expansion or thermal expansion		$Kelvin^{-1}$	$[M^0L^0T^0\theta^{-1}]$
(41)	Mechanical eq. of Heat (J)		Joule/Calorie	$[M^0L^0T^0]$
(42)	Vander wall's constant (a)		$Newton\text{-}metre^4$	$[ML^5T^{-2}]$
(43)	Vander wall's constant (b)		m^3	$[M^0L^3T^0]$

Electricity

S. N.	Quantity	Relation with other quantities	Unit	Dimension
(44)	Electric charge (q)	q=It	Coulomb	$[M^0L^0T^1A^1]$
(45)	Electric current (I)		Ampere	$[M^0L^0T^0A^1]$
(46)	Capacitance (C)	$C=\frac{q}{V}$	Coulomb/volt or Farad	$[M^{-1}L^{-2}T^4A^2]$
(47)	Electric potential (V)	$V=\frac{W}{q}$	Joule/coulomb	$M^1L^2T^{-3}A^{-1}$
(48)	Permittivity of free space (ε_0)	$\varepsilon_0=\frac{q_1q_2}{4\pi r^2F}$	$\frac{Coulomb^2}{Newton\text{-}meter^2}$	$[M^{-1}L^{-3}T^4A^2]$
(49)	Dielectric constant (K)		Unitless	$[M^0L^0T^0]$
(50)	Resistance (R)		Volt/Ampere or ohm	$[M^1L^2T^{-3}A^{-2}]$
(51)	Resistivity or Specific resistance (ρ)	$\rho=\frac{RA}{l}$	Ohm-meter	$[M^1L^3T^{-3}A^{-2}]$
(52)	Coefficient of Self-induction (L)	$L=\frac{\phi}{I}$	$\frac{volt-second}{ampere}$ or henry or ohm-second	$[M^1L^2T^{-2}A^{-2}]$
(53)	Magnetic flux (ϕ)	$\phi=BA$	Volt-second or weber	$[M^1L^2T^{-2}A^{-1}]$

S. N.	Quantity	Relation with other quantities	Unit	Dimension
(54)	Magnetic induction (B)	$B=\frac{F}{qv}$	$\frac{\text{newton}}{\text{ampere}-\text{meter}}$ $\frac{\text{Joule}}{\text{ampere}-\text{meter}^2}$ $\frac{\text{volt}-\text{second}}{\text{meter}^2}$ or Tesla	$[M^1L^0T^{-2}A^{-1}]$
(55)	Magnetic Intensity (H)		Ampere/meter	$[M^0L^{-1}T^0A^1]$
(56)	Magnetic Dipole Moment (M)		Ampere-meter^2	$[M^0L^2T^0A^1]$
(57)	Permeability of Free Space (μ_0)		$\frac{\text{Newton}}{\text{ampere}^2}$ or $\frac{\text{Joule}}{\text{ampere}^2-\text{meter}}$ or $\frac{\text{Volt}-\text{second}}{\text{ampere}-\text{meter}}$ or $\frac{\text{Ohm}-\text{second}}{\text{meter}}$ or $\frac{\text{henery}}{\text{meter}}$	$[M^1L^1T^{-2}A^{-2}]$
(58)	Surface charge density (σ)		Coulomb metre^{-2}	$[M^0L^{-2}T^1A^1]$
(59)	Electric dipole moment (p)	$P=q\times 2l$	Coulomb – meter	$[M^0L^1T^1A^1]$
(60)	Conductance (G)	$G=\frac{1}{R}$	ohm^{-1}	$[M^{-1}L^{-2}T^3A^2]$
(61)	Conductivity (σ)	$\sigma=\frac{1}{\rho}$	ohm^{-1}meter^{-1}	$[M^{-1}L^{-3}T^3A^2]$
(62)	Current density (J)		Ampere/m^2	$M^0L^{-2}T^0A^1$
(63)	Intensity of electric field (E)	$E=\frac{V}{l}$ or $\frac{F}{q}$	Volt/meter, Newton/coulomb	$M^1L^1T^{-3}A^{-1}$
(64)	Rydberg constant (R)		m^{-1}	$M^0L^{-1}T^0$

QUANTITIES HAVING SAME DIMENSIONS

	Dimension	Quantity
(1)	$[M^0L^0T^{-1}]$	Frequency, angular frequency, angular velocity, velocity

		gradient and decay constant
(2)	$[M^1L^2T^{-2}]$	Work, internal energy, potential energy, kinetic energy, torque, moment of force
(3)	$[M^1L^{-1}T^{-2}]$	Pressure, stress, Young's modulus, bulk modulus, modulus of rigidity, energy density
(4)	$[M^1L^1T^{-1}]$	Momentum, impulse
(5)	$[M^0L^1T^{-2}]$	Acceleration due to gravity, gravitational field intensity
(6)	$[M^1L^1T^{-2}]$	Thrust, force, weight, energy gradient
(7)	$[M^1L^2T^{-1}]$	Angular momentum and Planck's constant
(8)	$[M^1L^0T^{-2}]$	Surface tension, Surface energy (energy per unit area)
(9)	$[M^0L^0T^0]$	Strain, refractive index, relative density, angle, solid angle, distance gradient, relative permittivity (dielectric constant), relative permeability etc.
(10)	$[M^0L^2T^{-2}]$	Latent heat and gravitational potential
(11)	$[M^0L^2T^{-2}\theta^{-1}]$	Thermal capacity, gas constant, Boltzmann constant and entropy
(12)	$[M^0L^0T^1]$	$\sqrt{l/g}, \sqrt{m/k}, \sqrt{R/g}$, where l = length g = acceleration due to gravity, m = mass, k = spring constant
(13)	$[M^0L^0T^1]$	L/R, $\sqrt{LC}$, RC where L = inductance, R = resistance, C = capacitance
(14)	$[ML^2T^{-2}]$	$I^2Rt, \frac{V^2}{R}t$, VIt, qV, LI^2, $\frac{q^2}{C}, CV^2$ where I = current, t = time, q = charge, L = inductance, C = capacitance, R = resistance

APPLICATION OF DIMENSIONAL ANALYSIS

(1) **To find the unit of a physical quantity in a given system of units:** Writing the definition or formula for the physical quantity we find its dimensions. Now in the dimensional formula replacing M, L and T by the fundamental units of the required system we get the unit of physical quantity. However, sometimes to this unit we further assign a specific name, e.g., Work = Force × Displacement

So, $[W] = [MLT^{-2}] \times [L] = [ML^2T^{-2}]$

So, its units in C.G.S. system will be g cm^2/s^2 which is called erg while in M.K.S. system will be kg m^2/s^2 which is called joule.

(2) **To find dimensions of physical constant or coefficients**: As dimensions of a physical quantity are unique, we write any formula or equation incorporating the given constant and then by substituting the dimensional

formulae of all other quantities, we can find the dimensions of the required constant or coefficient.

(i) Gravitational constant: According to Newton's law of gravitation $F = G\frac{m_1 m_2}{r^2}$ or $G = \frac{Fr^2}{m_1 m_2}$

Substituting the dimensions of all physical quantities

$$[G] = \frac{[MLT^{-2}][L^2]}{[M][M]} = [M^{-1}L^3T^{-2}]$$

(ii) Plank constant: According to Planck $E = h\nu$ or $h = \frac{E}{\nu}$

Substituting the dimensions of all physical quantities

$$[h] = \frac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T^{-1}]$$

(iii) Coefficient of viscosity: According to Poiseuille's formula $\frac{dV}{dt} = \frac{\pi pr^4}{8\eta l}$ or

$$\eta = \frac{\pi pr^4}{8l(dV/dt)}$$

Substituting the dimensions of all physical quantities

$$[\eta] = \frac{[ML^{-1}T^{-2}][L^4]}{[L][L^3/T]} = [ML^{-1}T^{-1}]$$

(3) **To convert a physical quantity from one system to the other:** The measure of a physical quantity is nu = constant

If a physical quantity X has dimensional formula $[M^aL^bT^c]$ and if (derived) units of that physical quantity in two systems are $[M_1^aL_1^bT_1^c]$ and $[M_2^aL_2^bT_2^c]$ respectively and n_1 and n_2 be the numerical values in the two systems respectively, then $n_1[u_1] = n_2[u_2]$

$$\Rightarrow n_1[M_1^aL_1^bT_1^c] = n_2[M_2^aL_2^bT_2^c]$$

$$\Rightarrow n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

where M_1, L_1 and T_1 are fundamental units of mass, length and time in the first (known) system and M_2, L_2 and T_2 are fundamental units of mass, length and time in the second (unknown) system. Thus, knowing the values of fundamental units in two systems and numerical value in one system, the numerical value in other system may be evaluated.

Ex: (1) conversion of Newton into Dyne.

The Newton is the S.I. unit of force and has dimensional formula $[MLT^{-2}]$.

So, 1 N = 1 kg-m/sec^2

By using $n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c = 1\left[\frac{kg}{gm}\right]^1\left[\frac{m}{cm}\right]^1\left[\frac{sec}{sec}\right]^{-2}$

$= 1\left[\frac{10^3 gm}{gm}\right]^1\left[\frac{10^2 cm}{cm}\right]^1\left[\frac{sec}{sec}\right]^{-2} = 10^5$

$\therefore$ 1 N = 10^5 Dyne

(2) Conversion of gravitational constant (G) from C.G.S. to M.K.S. system

The value of G in C.G.S. system is 6.67×10^{-8} C.G.S. units while its dimensional formula is $[M^{-1}L^3T^{-2}]$

So, $G = 6.67 \times 10^{-8}$ cm^3/g s^2

By using $n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c = 6.67\times10^{-8}\left[\frac{gm}{kg}\right]^{-1}\left[\frac{cm}{m}\right]^3\left[\frac{sec}{sec}\right]^{-2}$

$= 6.67\times10^{-8}\left[\frac{gm}{10^3 gm}\right]^{-1}\left[\frac{cm}{10^2 cm}\right]^3\left[\frac{sec}{sec}\right]^{-2} = 6.67\times10^{-11}$

$\therefore$ G = 6.67×10^{-11} M.K.S. units

(4) **To check the dimensional correctness of a given physical relation:** This is based on the 'principle of homogeneity'. According to this principle the dimensions of each term on both sides of an equation must be the same.

If $X = A \pm (BC)^2 \pm \sqrt{DEF}$,

Then according to principle of homogeneity $[X] = [A] = [(BC)^2] = [\sqrt{DEF}]$

If the dimensions of each term on both sides are same, the equation is dimensionally correct, otherwise not. A dimensionally correct equation may or may not be physically correct.

Ex: (1) $F = mv^2 / r^2$

By substituting dimension of the physical quantities in the above relation

$[MLT^{-2}] = [M][LT^{-1}]^2 / [L]^2$

i.e., $[MLT^{-2}] = [MT^{-2}]$

As in the above equation dimensions of both sides are not same; this formula is not correct dimensionally, so can never be physically.

(2) $s = ut - (1/2)at^2$

By substituting dimension of the physical quantities in the above relation

$[L] = [LT^{-1}][T] - [LT^{-2}][T^2]$ i.e., $[L] = [L] - [L]$

As in the above equation dimensions of each term on both sides are same, so this equation is dimensionally correct. However, from equations of motion we know that $s = ut + (1/2)at^2$

2. DIMENSIONS OF A PHYSICAL QUANTITY

WORK SHEET LEVEL–I MAINS CORNER

SINGLE CORRECT ANSWER TYPE QUESTIONS

DIMENSIONAL FORMULA

1. Dimensional formula $\left[ML^2T^{-3}\right]$ represents

   1)Force 2)Power 3)Energy 4)Work

2. Whose dimensions is $\left[ML^2T^{-1}\right]$

   1)Torque 2)Angular momentum
   3)Power 4)Work

3. The dimensions of couple are

   1)$\left[ML^2T^{-2}\right]$ 2)$\left[MLT^{-2}\right]$ 3)$\left[ML^{-1}T^{-3}\right]$ 4)$\left[ML^{-2}T^{-2}\right]$

4. Dimensional formula for angular momentum is

   1)$\left[ML^2T^{-2}\right]$ 2)$\left[ML^2T^{-1}\right]$ 3)$\left[MLT^{-1}\right]$ 4)$\left[M^0L^2T^{-2}\right]$

5. Dimensional formula $\left[ML^{-1}T^{-2}\right]$ does not represent the physical quantity

   1)Young's modulus of elasticity 2)Stress
   3)Strain 4)Pressure

APPLICATIONS OF DIMENSIONAL ANALYSIS

6. Which one of the following does not have the same dimensions

   1)Work and energy 2)Angle and strain
   3)Relative density and refractive index
   4)Planck constant and energy

7. The foundations of dimensional analysis were laid down by

   1)Galileo 2)Newton 3)Fourier 4)Joule

8. The dimensions of stress are equal to

   1) Force 2)Pressure 3)Work 4) $\frac{1}{\text{Pressure}}$

9. If $x = at + bt^2$, where x is the distance travelled by the body in kilometre while t the time in seconds, then the units of b are

   1)km/s 2)km-s 3)km/s^2 4)km-s^2

LEVEL–II

DIMENSIONAL FORMULA

10. A force F is given by $F = at + bt^2$, where t is time. What are the dimensions of a and b

1) $\left[MLT^{-3}\right]$ and $\left[ML^2T^{-4}\right]$ 2) $\left[MLT^{-3}\right]$ and $\left[MLT^{-4}\right]$

3) $\left[MLT^{-1}\right]$ and $\left[MLT^{0}\right]$ 4) $\left[MLT^{-4}\right]$ and $\left[MLT^{1}\right]$

11. The dimensional formula of physical quantity X in the equation Force $= \frac{X}{\text{Density}}$ is given by

1) $\left[M^1L^4T^{-2}\right]$ 2) $\left[M^2L^{-2}T^{-1}\right]$ 3) $\left[M^2L^{-2}T^{-2}\right]$ 4) $\left[M^1L^{-2}T^{-1}\right]$

12. E, m, l and G denote energy, mass, angular momentum and gravitational constant respectively, then the dimension of $\frac{El^2}{m^5 G^2}$ are of

1) Angle 2) Length 3) Mass 4) Time

APPLICATIONS OF DIMENSIONAL ANALYSIS

13. In C.G.S. system the magnitude of the force is 100 dynes. In another system where the fundamental physical quantities are kilogram, metre and minute, the magnitude of the force is

1)0.036 2)0.36 3)3.6 4)36

14. Conversion of 1 MW power on a new system having basic units of mass, length and time as 10kg, 1dm and 1 minute respectively is

1) 2.16×10^{12} unit 2) 1.26×10^{12} unit 3) 2.16×10^{10} unit 4) 2×10^{14} unit

15. If 1gm cms^{-1} = x Ns, then number x is equivalent to

1) 1×10^{-1} 2) 3×10^{-2} 3) 6×10^{-4} 4) 1×10^{-5}

LEVEL-III ADVANCED CORNER

SINGLE CORRECT ANSWER TYPE QUESTIONS

16. From the dimensional consideration, which of the following equation is correct

1) $T = 2\pi\sqrt{\frac{R^3}{GM}}$ 2) $T = 2\pi\sqrt{\frac{GM}{R^3}}$ 3) $T = 2\pi\sqrt{\frac{GM}{R^2}}$ 4) $T = 2\pi\sqrt{\frac{R^2}{GM}}$

17. With the usual notations, the following equation $S_t = u + \frac{1}{2}a(2t-1)$ is
1)Only numerically correct
2)Only dimensionally correct
3)Both numerically and dimensionally correct
4)Neither numerically nor dimensionally correct

18. If velocity v, acceleration A and force F are chosen as fundamental quantities, then the dimensional formula of angular momentum in terms of v, A and F would be
1) $\left[FA^{-1}v\right]$ 2) $\left[Fv^3A^{-2}\right]$ 3) $\left[Fv^2A^{-1}\right]$ 4) $\left[F^2v^2A^{-1}\right]$

19. The largest mass (m) that can be moved by a flowing river depends on velocity (v), density (ρ) of river water and acceleration due to gravity (g). The correct relation is
1) $m \propto \frac{\rho^2 v^4}{g^2}$ 2) $m \propto \frac{\rho v^6}{g^2}$ 3) $m \propto \frac{\rho v^4}{g^3}$ 4) $m \propto \frac{\rho v^6}{g^3}$

20. A gas bubble from an explosion under water oscillates with a period T proportional to $P^a d^b E^c$, where P is the static pressure, d is the density and E is the total energy of the explosion. The values of a,b and c are:
1) a=0, b=1, c=2 2) a=1, b=2, c=3
3) a=5/6, b=-1/2, c=1/3 4) a=-5/6, b=1/2, c=1/3

21. A system has basic dimensions as density [D], velocity [V] and area [A]. The dimensional representation of force in this system is:
1) $[AV^2D]$ 2) $[A^2VD]$ 3) $[AVD^2]$ 4) $[A^0VD]$

22. According to Bernoulli's theorem, $\frac{P}{d} + \frac{v^2}{g} + gh = \text{constant}$,The dimensional formula of the constant is:(P-pressure, d-density, v-velocity, h-height, g-acceleration due to gravity]
1) $[M^0L^0T^0]$ 2) $[M^0LT^0]$ 3) $[M^0L^0T^{-2}]$ 4) $[M^0L^2T^{-2}]$

LEVEL–IV

STATEMENT TYPE QUESTIONS

23. Statement I: Linear mass density has the dimensions of $[M^1L^{-1}T^0]$.
Statement II: Density is defined as mass per unit volume.
1) Both statements are true
2) Both statements are false
3) Statement I is true, statement II is false
4) Statement I is false, statement II is true

MULTI CORRECT ANSWER TYPE QUESTIONS

24. Regarding dimension, which of the following statements are correct?
    1) The pure number are dimensionless
    2) A physical quantity that does not have any unit must be dimensionless
    3) The dimensional formula of force is $[MLT^2]$
    4) Strain is a dimensionless physical quantity

LEVEL–V

COMPREHENSION TYPE QUESTIONS

PASSAGE:

Consider Pressure P, velocity V and time T as fundamental (Base) quantities and answer the following questions.

25. The dimensional formula for force is
    1) $[PV^2T]$ 2) $[PVT^2]$ 3) $[PV^2T^2]$ 4) $[P^2VT]$
26. The physical quantity represented by dimensional formula PV^3T^2 is
    1) Work 2) Power 3) Momentum 4) Torque
27. If dimensional formula for physical quantity impulse (I=Ft) is expressed as $[I]=[P^aV^bT^c]$ then
    1) a=-1; b=2; c=3 2) a=1; b=-2; c=-3
    3) a=-1; b=-2; c=-3 4) a=1; b=2; c=3

MATRIX MATCH TYPE QUESTIONS

28. Match column I with column II and select the correct answer using the codes given below the lists:

COLUMN-I	COLUMN-II
a) Boltzmann constant	p) $[ML^2T^{-1}]$
b) Coefficient viscosity	q) $[ML^{-1}T^{-1}]$
c) Plank constant	r) $[MLT^{-3}K^{-1}]$
d) Thermal conductivity	s) $[ML^2T^{-2}K^{-1}]$

3. SIGNIFICANT FIGURES AND ERRORS

	♦	SIGINIFICANT FIGURES
	♦	ERRORS

SIGNIFICANT FIGURES

Significant figures in the measured value of a physical quantity tell the number of digits in which we have confidence. Larger the number of significant figures obtained in a measurement, greater is the accuracy of the measurement.

"All accurately known digits in a measurement plus the first uncertain digit together form significant figures."

For example, when we measure the length of a straight line using a meter scale and it lies between 7.4cm and 7.5cm, we may estimate it as l=7.43cm. This expression has three significant figures out of these 7and 4 precisely known but the last digit 3 is only approximately known.

RULES FOR COUNTING SIGNIFICANT FIGURES:

For counting significant figures, we use the following rules:

Rule 1: All non-zero digits are significant. For example x= 2567 has four significant figures.

Rule 2: The zeros appearing between two non-zero digits are counted in significant figures. For example 6.028 has 4 significant figures.

Rule 3: The zeros occurring to the left of last non-zero digit are NOT significant. For example0.0042 has two significant figures.

Rule 4: In a number without decimal, zeros to the right of non-zero digit are NOT significant. However when some value is recorded on the basic of actual measurement the zeros to the right of non-zero digit become significant. For example L=20m has two significant figures but x=200 has only one significant figure.

Rule 5: In a number with decimal, zeros to the right of last non-zero, digit are significant. For example x=1,400 has four significant figures.

Rule 6: The powers of ten are NOT counted as significant digits. For example 1.4×10^{-7} has only two significant figures 1 and 4.

Rule 7: Change in the units of measurement of a quantity does not change the number of significant figures. For example, suppose distance between two stations is 4067m. It has four significant figures. The same distance can be expressed as 4.067 km or $4.067\times10^{5}\,\text{cm}$. In all these expressions, number of significant figures continues to be four.

Table:

Measured value	Number of significant figures	Rule
127	3	1
572.5	4	2
0.06	1	3
530g	3	4
650	2	4
2.30	3	5
1.6×10^{14}	2	6

ROUNDING OFF A DIGIT:

Following are the rules for rounding off a measurement:

Rule 1: If the number lying to the right of cutoff digit is less than 5, then the cut off digit is retained as such. However, if it is more than 5, then the cut off digit is increased by 1. For example, x= 6.24 is rounded off to 6.2 to two significant digits and x=5.328 is rounded off to 5.33 to three significant digits.

Rule 2: If the digit to be dropped is 5 followed by digits other than zero then the preceding digit is increased by 1. For example =14.252 is rounded off to x=14.3 to three significant digits.

Rule 3: If the digit to be dropped is simply 5 or 5 followed by zeros, then the preceding digit is left unchanged if it is even. For example, x=6.250 or x=6.25 becomes x=6.2 after rounding off to two significant digits.

Rule 4: If the digit to be dropped is 5 or 5followed by zeros, then the preceding digit is raised by one if it is odd. For example x=6.350or x=6.35 becomes x=6.4 after rounding off to two significant digits.

Example:

Measured value	After rounding off to three significant digits	Rule
8.372	8.37	1
8.378	8.38	1
8.4281	8.43	2
8.445	8.44	3
8.4450	8.44	3
8.75	8.8	4
8.7500	8.8	4

ALGEBRAIC OPERATIONS WITH SIGNIFICANT FIGURES:

In addition, subtraction, multiplication or division inaccuracy in the measurement of any one variable affects the accuracy of the final result. Hence, in general, the final result shall have significant figures corresponding to their number in the least accurate variable involved.

To understand this, let us consider a chain of which all links are strong except the one. The chain will obviously break at the weakest link. Thus, the strength of the chain cannot be more than the strength of the weakest link in the chain. Hence, we do not imply a greater accuracy in our result than was obtained originally in our measurements.

i. Addition and subtraction

Suppose in the measured values to be added or subtracted the least number of significant digits after the decimal is n. Then in the sum or difference also, the number of significant digits after the decimal should be n.

Ex: 1.2 + 3.45+6.789=11.439 = 11.4

Here, the least number of significant digits after the decimal is one. Hence, the result will be 11.4(when rounded off to smallest number of decimal places).

Ex: 12.63-10.2=2.43 = 2.4

ii. Multiplication or division:

Suppose in the measured values to be multiplied or divided the least number of significant digits be n. Then in the product or quotient, the number of significant digits should also be n.

Ex: $1.2 \times 36.72=44.064 = 44$

The least number of significant digits in the measured values are two. Hence, the result when rounded off to two significant digits become 44. Therefore, the answer is 44.

Ex: 12.63-10.2=2.43 = 2.4

No measurement is perfect, as the errors involved in a measurement cannot be removed completely. Measured value is always somewhat different from the true value. The difference is called an error.

Errors can be classified in two ways. First classification is based on the cause of error. Systematic error and random errors fall in this group, Second Classification is based on the magnitude of error. Absolute error, mean absolute error and relative (or fractional) error lie on this group. Now, let us discuss them separately.

i. Systematic errors:

These are the errors whose cause are known tous. Such errors can therefore be minimized. Following are few causes of these errors:

a) Instrumental errors may be due to erroneous instruments. These errors can be reduced by using more accurate instruments and applying zero correction, when required.

b) Sometimes errors arise on account of ignoring certain facts. For example in measuring time period of simple pendulum error may creap because no consideration is taken of air resistance. These errors can be reduced by applying proper corrections to the formula used.

c) Change in temperature, pressure, humidity, etc., may also sometimes cause errors in the result. Relevant corrections can be made to minimize their effects.

ii. Random errors:

The causes of random errors are not known. Hence, it is not possible to remove them completely. These errors may arise due to a variety of reasons. For example the reading of a sensitive beam balance may change by the vibrations caused in the building due to persons moving in the laboratory or vehicles running near by. The random errors can be minimized by repeating the observation a large number of times and taking the arithmetic mean of all the observations. The mean value would be very close to the most accurate reading. Thus, $a_{mean} = \frac{a_1 + a_2 + ... + a_n}{n}$

iii. Absolute errors:

The difference between the true value and the measured value of a quantity is called an absolute error. Usually, the mean value a_m is taken as the true value.

So, if, $a_m = \frac{a_1 + a_2 + ... + a_n}{n}$

Then by definition, absolute errors in the measured values of the quantity are,

$\Delta a_1 = a_m - a_1$

$\Delta a_2 = a_m - a_2$

....

$\Delta a_n = a_m - a_n$

Absolute error may be positive or negative.

iv. Mean absolute error

Arithmetic mean of the magnitudes of absolute errors in all the measurements is called the mean absolute error. Thus, $\Delta a_{mean} = \frac{|\Delta a_1| + |\Delta a_2| + ... + |\Delta a_n|}{n}$

The final result of measurement can be written as,

$a = a_m \pm \Delta a_{mean}$

This implies that value of a is likely to lie between $a_m + \Delta a_{mean}$ and $a_m - \Delta a_{mean}$.

RELATIVE OR FRACTIONAL ERROR:

The ratio of mean absolute error to the mean value of the quantity measured is called relative or fractional error. Thus, $\text{Relative error} = \frac{\Delta a_{mean}}{a_m}$

Relative error expressed in percentage is called as the percentage error, i.e.,

$$\text{Percentage error} = \frac{\Delta a_{mean}}{a_m} \times 100$$

Examples:

1. The diameter of a wire as measured by screw gauge was found to be 2.620, 2.625, 2.630,2.628 and 2.626cm. Calculate:

a) mean value of diameter

b) absolute error in each measurement

c) mean absolute error000cm

c) Mean absolute error,

$$\Delta a_{mean} = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + |\Delta a_4| + |\Delta a_5|}{5}$$

$$= \frac{0.006 + 0.001 + 0.004 + 0.002 + 0.000}{5}$$

$= 0.0026 = 0.003$

(rounding off to three decimal places)

d) Fractional error

$$= \pm\frac{\Delta a_{mean}}{a_m} = \frac{\pm 0.003}{2.626} = \pm 0.001$$

e) Percentage error $= \pm 0.001 \times 100 = \pm 0.1\%$

f) Diameter of wire can be written as, $d = 2.626 \pm 0.1\%$

COMBINATION OF ERRORS:

i. Errors in sum or difference:

Let $x = a \pm b$

Further, let Δa is the absolute error in the measurement of a, Δb the absolute error in the measurement of b and Δx is the absolute error in the measurement of x.

Then,

$$x + \Delta x = (a \pm \Delta a) \pm (b \pm \Delta b)$$

$$= (a \pm b) \pm (\pm \Delta a \pm \Delta b)$$

$$= x \pm (\pm \Delta a \pm \Delta b)$$

or $\Delta x = \pm \Delta a \pm \Delta b$

The four possible values of Δx are ($\Delta a - \Delta b$),($\Delta a + \Delta b$), ($\Delta a - \Delta b$) and ($-\Delta a + \Delta b$).

Therefore, the maximum absolute error in x is, $\Delta x = \pm(\Delta a + \Delta b)$

i.e., the maximum absolute error in sum and difference of two quantities is equal to sum of the absolute errors in the individual quantities.

2. The volumes of two bodies are measured to be $V_1 = (10.2 \pm 0.02)\,cm^3$ and $V_2 = (6.4 \pm 0.01)\,cm^3$. Calculate sum and difference in volumes with error limits.

Sol. $V_1 = (10.2 \pm 0.02)\,cm^3$

$V_2 = (6.4 \pm 0.01)\,cm^3$

$\Delta V = \pm(\Delta V_1 + \Delta V_2)$

$= \pm(0.02 + 0.01)\,cm^3$

$= \pm 0.03\,cm^3$

$V_1 + V_2 = (10.2 + 6.4)\,cm^3 = 16.6\,cm^3$

and $V_1 - V_2 = (10.2 - 6.4)\,cm^3 = 3.8\,cm^3$

Hence, sum of volumes $= (16.6 \pm 0.03)\,cm^3$

and difference of volumes $= (3.8 \pm 0.03)\,cm^3$

ii. Errors in a product

Let x=ab

Then, $(x \pm \Delta x) = (a \pm \Delta a)(b \pm \Delta b)$

$$\text{or } x\left(1 \pm \frac{\Delta x}{x}\right) = ab\left(1 \pm \frac{\Delta a}{a}\right)\left(1 \pm \frac{\Delta b}{b}\right)$$

$$\text{or } 1 \pm \frac{\Delta x}{x} = 1 \pm \frac{\Delta b}{b} \pm \frac{\Delta a}{a} \pm \frac{\Delta a}{a}.\frac{\Delta b}{b}$$

$$\text{or } \pm \frac{\Delta x}{x} = \pm \frac{\Delta a}{a} \pm \frac{\Delta b}{b} \pm \frac{\Delta a}{a}.\frac{\Delta b}{b}$$

Here, $\frac{\Delta a}{a}.\frac{\Delta b}{b}$ is a small quantity, so can be neglected. Hence,

$$\pm \frac{\Delta x}{x} = \pm \frac{\Delta a}{a} \pm \frac{\Delta b}{b}$$

Possible values of $\frac{\Delta x}{x}$ are $\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$, $\left(\frac{\Delta a}{a} - \frac{\Delta b}{b}\right)$, $\left(-\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$ and $\left(-\frac{\Delta a}{a} - \frac{\Delta b}{b}\right)$

Hence, maximum possible value of $\frac{\Delta x}{x} = \pm\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$

Therefore, maximum fractional error in product of two (or more) quantities is equal to sum of fractional errors in the individual quantities.

iii. Errors in division

Let $x = \frac{a}{b}$

Then, $x \pm \Delta x = \frac{a \pm \Delta a}{b \pm \Delta b}$

or $x\left(1 \pm \frac{\Delta x}{x}\right) = \frac{a\left(1 \pm \frac{\Delta a}{a}\right)}{b\left(1 \pm \frac{\Delta b}{b}\right)}$

or $\left(1 \pm \frac{\Delta x}{x}\right) = \left(1 \pm \frac{\Delta a}{a}\right)\left(1 \pm \frac{\Delta b}{b}\right)^{-1}$

$\left[\text{as } x = \frac{a}{a}\right]$

As $\frac{\Delta b}{b} << 1$, so expanding binomially, we get

$$\left(1 \pm \frac{\Delta x}{x}\right) = \left(1 \pm \frac{\Delta a}{a}\right)\left(1 \pm \frac{\Delta b}{b}\right)$$

or $1 \pm \frac{\Delta x}{x} = 1 \pm \frac{\Delta x}{a} \pm \frac{\Delta b}{b} \pm \frac{\Delta a}{a}.\frac{\Delta b}{b}$

Here, $\frac{\Delta a}{a}.\frac{\Delta b}{b}$ is small quantity, so can be neglected. Therefore,

$$\pm \frac{\Delta x}{x} = \pm \frac{\Delta x}{a} \pm \frac{\Delta b}{b}$$

Possible values of $\frac{\Delta x}{x}$ are $\left(\frac{\Delta a}{a} - \frac{\Delta b}{b}\right)$, $\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$, $\left(-\frac{\Delta a}{a} - \frac{\Delta b}{b}\right)$ and $\left(-\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$.

Therefore, the maximum value of

$$\frac{\Delta x}{x} = \pm\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$$

or, the maximum value of fractional error indivision of two quantities is equal to the sum of fractional errors in the individual quantities.

iv. Error in quantity raised to some power

Let $x = \frac{a^n}{b^m}$

Then, ln(x)=n ln(a)-m ln(b)

Differentiating both sides, we get

$$\frac{dx}{x} = n.\frac{da}{a} - m\frac{db}{b}$$

In terms of fractional error we may write,

$$\pm\frac{\Delta x}{x} = \pm n\frac{\Delta a}{a} \mp m\frac{\Delta b}{b}$$

Therefore, maximum value of

$$\frac{\Delta x}{x} = \pm\left(n\frac{\Delta a}{a} + m\frac{\Delta b}{b}\right)$$

3. The mass and density of a solid sphere are measured to be (12.4 ± 0.1)kg and (4.6 ± 0.2)kg / m^3. Calculate the volume of the sphere with error limits.

Sol. Here, $m \pm \Delta m = (12.4 \pm 0.1)$kg

and $\rho \pm \Delta\rho = (4.6 \pm 0.2)$kg / m^3

$$\text{volume } V = \frac{m}{\rho} = \frac{12.4}{4.6} = 2.69m^3 = 2.7m^3$$

(rounding off to one decimal place)

$$\text{Now, } \frac{\Delta V}{V} = \pm\left(\frac{\Delta m}{m} + \frac{\Delta\rho}{\rho}\right)$$

$$\text{or } \Delta V = \pm\left(\frac{\Delta m}{m} + \frac{\Delta\rho}{\rho}\right) \times V$$

$$= \pm\left(\frac{0.1}{12.4} + \frac{0.2}{4.6}\right) \times 2.7 = \pm 0.14$$

$$= V \pm \Delta V = (2.7 \pm 0.14)m^3$$

4. Calculate percentage error in determination of time period of a pendulum.

$T = 2\pi\sqrt{\frac{l}{g}}$ where l and g are measured with $\pm 1\%$ and $\pm 2\%$ errors.

Sol. $$\frac{\Delta T}{T} \times 100 = \pm\left(\frac{1}{2} \times \frac{\Delta l}{l} \times 100 + \frac{1}{2} \times \frac{\Delta g}{g} \times 100\right)$$

$$= \pm\left(\frac{1}{2} \times 1 + \frac{1}{2} \times 2\right) = \pm 1.5\%$$

3. SIGNIFICANT FIGURES AND ERRORS

WORK SHEET LEVEL–I MAINS CORNER

SINGLE CORRECT ANSWER TYPE QUESTIONS

SIGNIFICANT FIGURE

1. Each side a cube is measured to be 7.203 m. The volume of the cube up to appropriate significant figures is

1)373.714 2)373.71 3)373.7 4)373

2. The number of significant figures in 0.007 m^2 is

1)1 2)2 3)3 4)4

3. The mass of a box is 2.3 kg. Two marbles of masses 2.15 g and 12.39 g are added to it. The total mass of the box to the correct number of significant figures is

1)2.340 kg 2)2.3145 kg 3)2.3 kg 4)2.31 kg

4. The length of a rectangular sheet is 1.5 cm and breadth is 1.203 cm. The area of the face of rectangular sheet to the correct no. of significant figures is:

1)1.8045 cm^2 2)1.804 cm^2 3)1.805 cm^2 4)1.8 cm^2

ERRORS

5. Zero error in an instrument introduces

1) Systematic error 2) Random error
3) Least count error 4) Personal error

6. The pressure on a square plate is measured by measuring the force on the plate and the length of the sides of the plate. If the maximum error in the measurement of force and length are respectively 4% and 2%, The maximum error in the measurement of pressure is

1)1% 2)2% 3)6% 4)8%

7. The resistance $R = \frac{V}{i}$ where V= 100 $\pm$ 5 volts and i = 10 $\pm$ 0.2 amperes. What is the total error in R

1)5% 2)7% 3)5.2% 4)$\frac{5}{2}$%

8. The period of oscillation of a simple pendulum in the experiment is recorded as 2.63 s, 2.56 s, 2.42 s, 2.71 s and 2.80 s respectively. The average absolute error is

1)0.1 s 2)0.11 s 3)0.01 s 4)1.0 s

LEVEL–II

SIGNIFICANT FIGURE

9. The number of significant figures in a pure number 410 is

1) Two 2) Three 3) One 4) Infinite

10. The number of significant figures in the measured value 26000 is
1) Five 2) Two 3) Three 4) Infinite

11. The number of significant figures in the measured value 0.0204 is
1) Five 2) Three 3) Four 4) Two

12. The number of significant zeroes present in the measured value 0.020040, is
1) Five 2) Two 3) One 4) Three

13. The number of significant figures in the measured value 4.700 m is the same as that in the value
1) 4700 m 2) 0.047 m 3) 4070 m 4) 470.0 m

ERRORS

14. If there is a positive error of 50% in the measurement of velocity of a body, then the error in the measurement of kinetic energy is
1)25% 2)50% 3)100% 4)125%

15. The percentage errors in the measurement of mass and speed are 2% and 3% respectively. How much will be the maximum error in the estimation of the kinetic energy obtained by measuring mass and speed
1)11% 2)8% 3)5% 4)1%

16. The mean time period of second's pendulum is 2.00s and mean absolute error in the time period is 0.05s. To express maximum estimate of error, the time period should be written as
1) (2.00 ± 0.01) s 2) $(2.00 + 0.025)$ s 3) (2.00 ± 0.05) s 4) (2.00 ± 0.10) s

17. A body travels uniformly a distance of (13.8 ± 0.2) m in a time (4.0 ± 0.3) s. The velocity of the body within error limits is
1) $(3.45 \pm 0.2)\ ms^{-1}$ 2) $(3.45 \pm 0.3)\ ms^{-1}$
3) $(3.45 \pm 0.4)\ ms^{-1}$ 4) $(3.45 \pm 0.5)\ ms^{-1}$

18. The percentage error in the above problem is
1)7% 2)5.95% 3)8.95% 4)9.85%

LEVEL-III ADVANCED CORNER

SINGLE CORRECT ANSWER TYPE QUESTIONS

19. The radius of a sphere is (5.3 ± 0.1) cm. The percentage error in its volume is
1) $\frac{0.1}{5.3}\times 100$ 2) $3\times\frac{0.1}{5.3}\times 100$ 3) $\frac{0.1\times 100}{3.53}$ 4) $3+\frac{0.1}{5.3}\times 100$

20. Calculate percentage error in the determination of $g = 4\pi^2\frac{L}{T^2}$ when 'L' and 'T' are measured with $\pm 2\%$ and $\pm 3\%$ error is respectively.
1) $\pm 6\%$ 2) $\pm 9\%$ 3) $\pm 8\%$ 4) $\pm 5\%$

21. What is error in density of a cube when its mass is uncertainty by $\pm 2\%$ and length of its edge is uncertainty by $\pm 1\%$?

1) $\pm 5\%$ 2) $\pm 8\%$ 3) $\pm 9\%$ 4) $\pm 6\%$

22. A physical quantity X is related to four measurable quantities a, b, c and d as follows: $X = a^2 b^3 c^{5/2} d^{-2}$. The percentage error in the measurement of a,b,c and d are 1%,2%, 3% and 4%. What is the percentage error in X?

1) 24.6% 2) 23.5% 3) 27.6% 4) 25.5%

23. The error in measurement of radius of a sphere is 1%. What is the error in the measurement of volume?

1) 2% 2) 4% 3) 1% 4) 3%

LEVEL–IV

STATEMENT TYPE QUESTIONS

24. Statement I: Absolute error is the difference between the measured value and the true value.

Statement II: Absolute error is always a negative quantity.

1) Both statements are true
2) Both statements are false
3) Statement I is true, statement II is false
4) Statement I is false, statement II is true

INTEGER TYPE QUESTIONS

25. The number of the significant figures in 11.118×10^{-6}V is _____

MULTI CORRECT ANSWER TYPE QUESTIONS

26. A stick has a length of 12.132cm and another stick a length of 12.4cm. Which of the following are true due regard to significant figures.

1) Total length is 24.532cm, if two sticks are placed end to end.
2) The difference in their lengths is 0.3cm, if two sticks are placed side by side.
3) Total length is 24.5cm, if two sticks are placed end to end.
4) The difference in their lengths is 0.268cm, if sticks are placed side by side.

27. Which of the following are true?

1) When two quantities are added or subtracted, the maximum absolute error in the result (in both cases) will be the sum of the absolute errors in two quantities.
2) When two quantities are multiplied or divided, the maximum absolute error in the result(in both cases) will be the sum of the absolute errors in the two quantities.

3) If $x = \frac{a^p b^q}{c^r}$ then $\frac{\Delta x}{x} = p\left(\frac{\Delta a}{a}\right) + q\left(\frac{\Delta b}{b}\right) + r\left(\frac{\Delta c}{c}\right)$

4) The final result of arithmetic operations should never have more significant figures than the least number of significant figures in the original components.

LEVEL–V

COMPREHENSION TYPE QUESTIONS

PASSAGE:

Suppose in the measured values to be added or subtracted the least number of significant digits after the decimal is n. Then in the sum or difference also, the number of significant digits after the decimal should be n.

Suppose in the measured values to be multiplied or divided the least number of significant digits be n. Then in the product or quotient, the number of significant digits should also be n. For square roots it is customary to have the number of significant figures one less than the number.

28. 124.2+52.487=________________

1) 176.69 2) 176.6 3) 176.7 4) 176.9

29. 117.3 $\times$ 0.0024=____________________

1) 0.2808 2) 0.281 3) 0.29 4) 0.28

30. 9.27 /41=__________

1) 0.226 2) 0.23 3) 0.22609 4) 0.2260975

MATRIX MATCH TYPE QUESTIONS

31. Match the numbers in column-I to the number of significant figures in column-II

Column-I	**Column-II**
a) 6729	p) three
b) 0.024	q) five
c) 0.08240	r) four
d) 2520×10^7	s) two
	t) seven